Ejercicio: Dado el espacio vectorial $\mathbb{C}^4$, se considera la aplicación:

f: $\mathbb{C}^4 \longrightarrow P_3(\mathbb{C})$ dada por $f(a, b, c, d) = (a - b) + cx + dx^2$

a) Calcular la matriz asociada a f respecto de las bases canónicas

b) ¿Es f diagonalizable? Razonar la respuesta.

a) $B_{\mathbb{C}^4} = \{(1,0,0,0), (0,1,0,0), (0,0,1,0), (0,0,0,1)\}$

$B_{P_3(\mathbb{C})} = \{1, x, x^2, x^3\}$

$f(1,0,0,0) = 1 = (1,0,0,0) = 1.(1) + 0.x + 0.x^2 + 0.x^3$

$f(0,1,0,0) = -1 = (-1,0,0,0)$

$f(0,0,1,0) = x = (0,1,0,0) = 0.1 + 1.x + 0.x^2 + 0.x^3$

$f(0,0,0,1) = x^2 = (0,0,1,0)$

$$A = M_{B_c B_c}(f) = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

b) $$p(\lambda) = |A - \lambda I| = \begin{vmatrix} 1-\lambda & -1 & 0 & 0 \\ 0 & -\lambda & 1 & 0 \\ 0 & 0 & -\lambda & 1 \\ 0 & 0 & 0 & -\lambda \end{vmatrix} = (1-\lambda)(-\lambda)^3 = -\lambda^3(1-\lambda)$$

$p(\lambda) = 0 \Rightarrow \lambda_1 = 1 \longrightarrow \alpha_1 = \underline{1 = d_1}$ $(1 \leq d_i \leq \alpha_i)$

$\lambda_2 = 0 \longrightarrow \alpha_2 = 3$

$$d_2 = n - rg(A - \cancel{0 \cdot I}) = 4 - rg\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} = 4 - 3 = 1$$

$$\alpha_2 = 3 \neq 1 = d_2$$

f no es diagonalizable.